

Numerical Methods with MS Excel

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Abstract: In this note we show how MS Excel can be used to perform numerical Integration, specifically Trapezoidal Rule and Simson's rule. Futhermore, we illustrate how to generate Lagranges Interpolation polynomial.

1. Introduction

MS Excel is the most commonly used spreadsheet, and has now grown into powerful software that can be used virtually by all branches of science and engineering. The availability of the program in almost all PCs makes its usage appears to be at the increase. The program has been use in teaching and solving many mathematical problems in many different ways. And this usage ranges from lower level to advanced level courses. The nature of the program makes numerical methods much easier to be implemented. However, since in Excel the users assume more responsibility in designing the application, and are in full control of the implementation, the program requires certain degree of creativity.

In this paper, we intend to illustrate how teachers and students can use Excel to implement three well-known numerical methods: Simpson's Rule, Trapezoidal Rules and Lagrange's interpolation. It should be noted that using Excel in addressing the first two integration techniques is not new (see references), however, our approach is much simpler, more direct, and do not require any *macros* for execution. In addition, our worksheet needs no modifications ones is developed, except for entering the interval of integration and the number of divisions (≤ 100). This gives us motivation to share our experience with larger mathematics education community.

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The motivation for Lagrange's interpolation stems from a question asked to Dr. Math in MathForum.org (<http://mathforum.org/library/drmath/view/63984.html>). The questioner wanted to know among other things how to use Excel to work out Lagrange's interpolation. Although the two other questions were answered sufficiently, the Excel implementation was skipped by Dr. Math. So in this note, we plan to answer that question as well as share our experience with larger mathematics community.

2. Numerical Integration with Excel

In this section we illustrate how Excel worksheets can be used to implement the trapezoidal rule and the Simpson's rule for numerical integration. The problem is to find a numerical approximation for the integral

$$I = \int_a^b f(x)dx .$$

2.1 The trapezoidal rule

The trapezoidal rule works by approximating the function $f(x)$ by a piecewise linear function and evaluate the integral of each piece. If the interval $[a, b]$ is divided up into n equal subinterval, each of width $h = \frac{b-a}{n}$, then the approximate integral is

$$I \approx \frac{h}{2} \sum_{i=1}^n f(x_{i-1}) + f(x_i), \text{ where, } x_i = a + ih, \text{ and } i = 0, 1, \dots, n$$

We illustrate with the following example

$$\int_0^1 (14x^6 + 7)dx.$$

The method is explained as follows. The endpoints (initial and terminal) of the interval, and the number of divisions are entered in the cells A2, B2, C2, respectively. The value of h is calculated in the cell D2 by entering the formula $= (B2-A2)/C2$.

To generate the x_i s we take the following steps:

1. In cell E2 enter the formula $=A2$. This copies the value of $a = x_0$ into E2. The following figure shows the upper part of the Excel worksheet implementation of the method.

	A	B	C	D	E	F	G	H
1	a	b	n	h	x_i	$f(x_i)$	A_i	
2	0	1	50	0.02	0	7		
3					0.02	7	0.14	
4					0.04	7	0.14	
5					0.06	7	0.14	
6					0.08	7	0.14	
7					0.1	7	0.14	
8					0.12	7	0.14	
9					0.14	7	0.14	
10					0.16	7	0.14	
11					0.18	7	0.14	
12					0.2	7.001	0.14	
13					0.22	7.002	0.14	
14					0.24	7.003	0.14	
15					0.26	7.004	0.14	
16					0.28	7.007	0.14	

2. The next values are generated with the formula $=IF(E2>= \$B\$2, \$B\$2, E2+ \$D\$2)$ in E3. This formula adds h to the previous value until we reach the value of b . Afterwards it keeps entering the value of b . This mechanism is used to enable changing the value of n to get more control on the accuracy of the solution as explained in Step 5.
3. Copy this formula to the next 100 cells or so below E4. The below figure shows a part of the sheet further down, where you can see the value of b being repeated.
4. The function f is entered in the column labeled $f(x_i)$ by entering the formula $=7+14 * E2^6$ in the cell F2 and copying it along the corresponding cells for the x_i s.
5. We then form the elements of the summation in the trapezoidal rule by entering the formula $= (E3-E2)/2*(F2+F3)$ in cell G3 and copying it along the corresponding cells for the x_i s. Observe that, instead of using the value of h generated in cell D2, we used the equivalent difference $E3-E2$. This has two advantages:
 - a) The formula produces zeros when we go past the right endpoint b . In this way, the final sum of these numbers is not affected by the repetition of b .
 - b) It allows the use of the trapezoidal rule with non-uniform divisions of the interval $[a, b]$.
6. The last step is to add the terms in column G to get the approximation of the integral. Select the range of cells that contains the summation terms and then click the sum button (Σ) on the toolbar. The result is shown below.

	A	B	C	D	E	F	G	H	I
96					1	21	0		
97					1	21	0		
98					1	21	0		
99					1	21	0		
100					1	21	0		
101					1	21	0		
102					1	21	0		
103							9.003		

Note: (a) If you now change the number of divisions n to 100, the new, more accurate approximation will appear in the same cell (G103).

(b) To change the interval of integration all you need to do is to change the values a, b in cells A2, B2.

(c) To change the integrated function enter the new formula in cell F2 and copy it to cell F103.

2.2 Simpson's rule

Simpson's rule finds an approximation of the value of the integral I by approximating the integrand with a piecewise polynomial of degree 2 and then evaluate the integral over each piece. Simpson's formula is

$$I \approx \frac{h}{3} \sum_{i=1}^{n-1} [f(x_{i-1}) + 4f(x_i) + f(x_{i+1}))],$$

Where the interval $[a, b]$ is divided into n intervals, each of length $h = \frac{b-a}{n}$. The Excel implementation of Simpson's rule is very much similar to that of the trapezoidal rule, except for some details. The following figure shows the upper part of the worksheet implementation.

	A	B	C	D	E	F	G	H	I
1	a	b	n	h	x_i	$f(x_i)$	S_i		
2	0	1	40	0.03	0	7			
3					0.03	7	0.35		
4					0.05	7			
5					0.08	7	0.35		
6					0.1	7			
7					0.13	7	0.35		
8					0.15	7			
9					0.18	7	0.35		
10					0.2	7			
11					0.23	7	0.35		
12					0.25	7			
13					0.28	7.01	0.35		
14					0.3	7.01			
15					0.33	7.02	0.35		
16					0.35	7.03			

As you can see from this figure the table is exactly the same as that for the trapezoidal rule except for the last column. The last column is generated as follows. In the cell G3 enter the formula

$$=(E4-E2)/6*(F2+4*F3+F4).$$

Since Simpson's formula spans two subintervals for each entry in the summation, the copying of the formula is done as follows. Select the range of two cells G2, G3 (note that G2 is actually empty). Using the formula copying technique, drag the two cells down to cell G103. The result is that the formula is copied to every other cell. One more thing to notice here is that, in the above formula, the value of h is replaced by the difference over 3 cells divided by 2. This way the same skipping is achieved and no problem arises as a result of repeating the values of b . When you select the range G2:G103 and click the sum button, you will see the result 9.000003644, which is more accurate than the result of the trapezoidal rule as the theory predicts.

3. Lagrange Interpolation

The idea of interpolation as succinctly elaborated in the site (<http://mathforum.org/library/drmath/view/63984.html>) is to find a function of a specified form which passes through a given list of points. Lagrange interpolation uses polynomials. For instance, it is well known that given two points (x_1, y_1) and (x_2, y_2) , there is exactly one line (degree one polynomial) that can be generated from these two points. It is also true that given three distinct points, there are two possibilities, either the three points lie in a line, or else there is exactly one quadratic polynomial that the three points pass through. If you have four points, there will be a cubic (at most) polynomial through them; if you have 10 points, there will be a polynomial of degree at most 9 through them. In general, given a list of n distinct points $\{(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots, (x_n, y_n)\}$, there will be a polynomial of degree at most

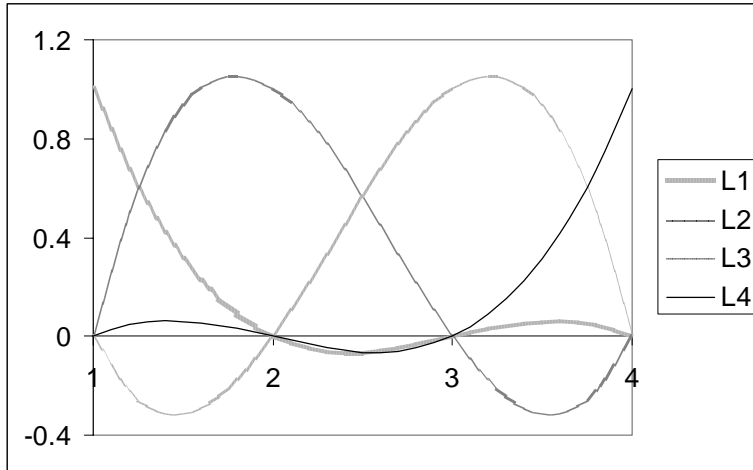
($n-1$) passing through the given points. In this section, we illustrate how one can use Excel to generate this polynomial. The steps are as follows:

1. Mark a range (say R(2,3):R(2,n+2)) as $x_1, x_2, x_3, \dots, x_n$ (this is a row of n cells starting at C2).
2. Mark a column (say A4:An+3 (R(4,1):R(n+3,1))) as $x_1, x_2, x_3, \dots, x_n$.
3. In the range R(3,3):R(3,n+2) enter the values of the interpolation points x_k from the interval $[a, b]$.
4. In the column B4:Bn+3, that is (R(4,2):R(n+3,2)) enter the values of the interpolation points x_k from the interval $[a, b]$.
5. In the cell C4 (i.e., R(4,3)) enter the formula = IF(C\$3-\$B4<>0;C\$3-\$B4;1). Notice that row 3 and column B are fixed in this formula.
6. Copy the formula through the range R(4,3):R(n+3,n+2). This should produce the differences $(x_i - x_j)$ for all i, j but replacing the difference $x_j - x_j$ by one instead of its true value 0. The net effect is to cancel the calculation of $x_j - x_j$.

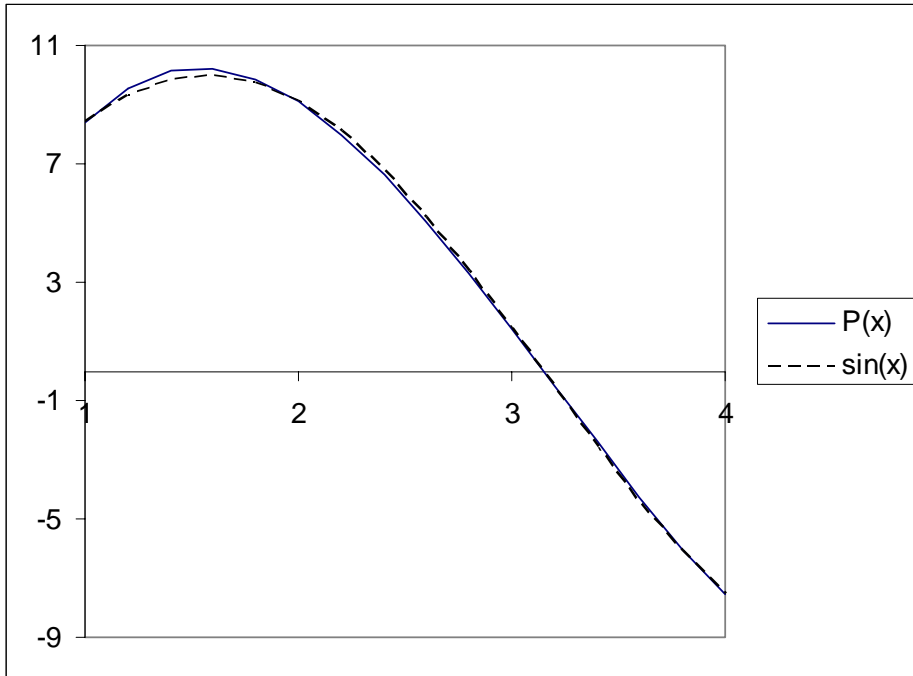
Interpolation points		x1	x2	x3	x4	products (x-xj) ->
		1	2	3	4	
x1	1	1	1	2	3	
x2	2	-1	1	1	2	
x3	3	-2	-1	1	1	
x4	4	-3	-2	-1	1	
Prod(xj-xi) ->		-6	2	-2	6	
x values	1	0	-1	-2	-3	0
	1.2	0.2	-0.8	-1.8	-2.8	-0.8064
	1.4	0.4	-0.6	-1.6	-2.6	-0.9984
	1.6	0.6	-0.4	-1.4	-2.4	-0.8064
	1.8	0.8	-0.2	-1.2	-2.2	-0.4224
	2	1	0	-1	-2	0
	2.2	1.2	0.2	-0.8	-1.8	0.3456
	2.4	1.4	0.4	-0.6	-1.6	0.5376
	2.6	1.6	0.6	-0.4	-1.4	0.5376
	2.8	1.8	0.8	-0.2	-1.2	0.3456
	3	2	1	0	-1	0
	3.2	2.2	1.2	0.2	-0.8	-0.4224
	3.4	2.4	1.4	0.4	-0.6	-0.8064
	3.6	2.6	1.6	0.6	-0.4	-0.9984
	3.8	2.8	1.8	0.8	-0.2	-0.8064
	4	3	2	1	0	0

7. In the cell R(n+4,3) enter the formula = Product(R(4,3):R(n+3,3)). The selection of the argument of the product sheet function can be done by dragging with the mouse. Copy the formula in R(n+4,3) through the range R(n+4,3): R(n+4,n+2) by dragging with the mouse. This gives the products that go in the denominators of the Lagrange basis functions.
8. Generate the set of x values that you want to compute at in the range R(n+5,2):R(n+m+5,2) from the interval $[a, b]$ by any method of sequence generation.
9. In the cell R(n+5,3) enter the formula =R(n+5,\$2)-R(\$3,3). Drag this formula through the range R(n+5,3):R(n+m+5,n+2). this generates a table of the values of $(x - x_i)$, $1 \leq i \leq n$.
10. In the cell R(n+5,n+3) enter the formula = Product(R(n+5,3):R(n+5,n+2). Drag this formula through the range R(n+5,n+3):R(n+m+5,n+3).
11. Now we are ready to generate the values of the Lagrange basis functions at the points x chosen in step 7 . Label the cells (should be blank) R(n+4,n+4):R(n+4,2n+3) as L_1, L_2, \dots, L_n .

L1	L2	L3	L4	f(xi)	P(x)	sin(x)
1	0	0	0	8.41471	8.41471	8.41471
0.672	0.504	-0.224	0.048	9.092974	9.55817	9.320391
0.416	0.832	-0.312	0.064	1.4112	10.14123	9.854497
0.224	1.008	-0.288	0.056	-7.56802	10.22038	9.995736
0.088	1.056	-0.176	0.032		9.852127	9.738476
0	1	0	0		9.092974	9.092974
-0.048	0.864	0.216	-0.032		7.99942	8.084964
-0.064	0.672	0.448	-0.056		6.627964	6.754632
-0.056	0.448	0.672	-0.064		5.035109	5.155014
-0.032	0.216	0.864	-0.048		3.277354	3.349882
0	0	1	0		1.4112	1.4112
0.032	-0.176	1.056	0.088		-0.50685	-0.58374
0.056	-0.288	1.008	0.224		-2.4203	-2.55541
0.064	-0.312	0.832	0.416		-4.27265	-4.4252
0.048	-0.224	0.504	0.672		-6.00739	-6.11858
0	0	0	1		-7.56802	-7.56802



12. In the cell $R(n+5,n+4)$ enter the formula $\text{If}(R(n+5,3)=0;1;R(n+5,\$n+3)/R(n+5,3)/R(\$n+4,3))$. Drag this formula through the range cell $R(n+5,n+4):R(n+m+5,2n+m+4)$. This generates the values of the Lagrange basis functions at the m chosen points x .
13. The final stage is to generate the Lagrange polynomial. Label the cells $R(n+4,2n+4)$ and $R(n+4,2n+5)$ as $f(x_i)$ and $P(x)$ respectively.
14. In the range $R(n+5,2n+4):R(2n+4:2n+4)$ enter the values of $f(x_i)$, $i = 1, 2, \dots, n$.
15. Select the range of cells (empty by now) $R(n+5,2n+5):R(n+m+5,2n+5)$ and enter the formula $=\text{MMULT}(R(n+5,n+4):R(n+m+5,2n+m+4); R(n+5,2n+4):R(2n+4:2n+4))$. This produces the values of $P(x)$ at the m points x .
16. The values of the original function can be produced next and then $f(x_i)$ and $P(x)$ can be plotted against x on the same graph.



References

<http://oregonstate.edu/instruct/ch490/lessons/lesson11.htm>

<http://oregonstate.edu/~haggertr/487/integrate.htm>

<http://www.math.villanova.edu/connections/Excel/>

<http://mathforum.org/library/drmath/view/63984.html>